Reserve Uncertainty and Speculative Attacks on Target Zones

Giulio Fella[*]

Queen Mary, University of London, Mile End Road, London E1 4NS, UK

March 2000

Abstract

Contrary to a peg, the sustainability of a currency band is enhanced by uncertainty about the availability of a secondary reserve. Furthermore, the critical size of reserves necessary to support a target zone is a decreasing function of the band upper boundary.

JEL Classification: F31, F32

Keywords: Target zones; Speculative attacks.

1 Introduction

It is a well known result in the literature on speculative attacks on fixed exchange rate regimes that uncertainty about the availability of a secondary reserve does not affect the timing and magnitude of an attack. As argued in Krugman (1979), provided the monetary authority is credibly committed to defend the peg while reserves last, an attack aimed at testing the availability of a secondary reserve is a one-sided bet. The exchange rate does not appreciate if the peg is still viable after the attack, but speculators would make foreseeable losses if they forwent the attack. Krugman (1997) has recently conjectured

---

that the reason why we do not observe speculative attacks at the minimum hint of trouble may be due to transaction costs or other microeconomic frictions.

This paper provides an additional explanation which applies to target zones. It shows that, unlike a peg, uncertainty about the level of reserves committed to the defense of a currency band increases its sustainability. In fact, attacking a target zone is not a one-sided bet. Demand for real money balances increases and the exchange rate appreciates to maintain money market equilibrium, if a secondary reserve turns out to be available. This risk of appreciation induces speculators to postpone the attack.

We also show that, as the intervention parity increases, the expected loss from carrying out an attack when the monetary authority can still defend the band in the post-attack scenario increases faster than the expected loss from foregoing the attack. This implies that, in the presence of reserve uncertainty and only in such a case, the critical reserve size is a decreasing function of the band upper boundary.

2 The model

Assume that the exchange rate is described by the simple log-linear monetary model

$$s = m + v + \alpha \frac{E(ds)}{dt}, \tag{1}$$

where s is the logarithm of the price of foreign exchange, α is a positive parameter and $E(.)$ is the expectation operator. $m = \ln(D + R)$ is the logarithm of the money supply, with D domestic credit, assumed to be constant, and R the stock of foreign currency reserves. v is a velocity shock (in logs) following a Brownian motion $dv = \sigma dz$ where σ is a positive constant and z a Wiener process. We normalize the initial value of m to zero and, for simplicity, follow Krugman and Rotemberg (1992) in assuming a one-sided band with upper boundary $\bar{s}$.

The monetary authority unilaterally defends the zone by infinitesimal marginal intervention as long as it has enough reserves. So the composite fundamental $m + v$ follows a regulated Brownian motion $d(m + v) = \sigma dz - dU$ where U is a continuous, non-negative, non-decreasing process, increasing only as long as the zone is in place and $s = \bar{s}$. We

normalize the lower bound on the primary stock of reserves to zero. Unless a secondary reserve is available, the central bank has to abandon intervention and float the currency as soon as $R = 0$.

The general solution to the functional equation (1) under our assumptions is given by

$$s = m + v + Ae^{\lambda(m+v)}, \tag{2}$$

where $\lambda = \sqrt{2/\alpha\sigma^2}$ and A is a constant to be determined. Assuming the initial stock of reserves is sufficient to support the band and that the monetary authority commitment is credible implies that the exchange rate path has to paste smoothly to the boundary of the band. This uniquely determines the particular solution

$$s = m + v - \frac{e^{\lambda[m+v-(\overline{m+v})]}}{\lambda}, \tag{3}$$

where $\overline{m+v}$ is the upper bound on the composite fundamental and satisfies the value matching condition

$$\bar{s} = \overline{m+v} - \frac{1}{\lambda}. \tag{4}$$

Once reserves have been exhausted, $m = \ln D$ and the exchange rate floats freely according to

$$s = \ln D + v. \tag{5}$$

Since the monetary authority intervenes only when $s = \bar{s}$, a speculative attack can only take place at the zone's boundary. Suppose that, with probability one, no secondary reserve is available. Then, the band collapses under an attack as soon as the shadow exchange rate given by equation (5) exceeds $\bar{s}$, that is for any level of the velocity shock above

$$v^* = \bar{s} - \ln D. \tag{6}$$

If this were not the case the exchange rate would jump in a foreseen way.

Since v is a continuous variable, the size of the speculative attack can be obtained by evaluating equation (5) at $\bar{s}$ and equating it to (4). Noticing that $m = \ln(D + R)$ before

3

the collapse, the discrete fall in reserves is implicitly given by

$$\ln(D + R^*) - \ln D = \frac{1}{\lambda}. \tag{7}$$

The quantity R^*, which is independent from $\bar{s}$, is the minimum amount of reserves consistent with a credible band. Only if the initial stock of reserves is strictly larger than R^*, is the expected survival time of the band strictly positive if the system is started at $s = \bar{s}$. This is illustrated in figure 1, where the TT curve is the exchange rate locus when reserves have reached the critical amount R^* and the $F'F'$ line is the post-collapse free-float locus described by equation (5). If the exchange rate is at its upper bound $\bar{s}$, the band collapses under a speculative attack which exhausts the remaining reserves R^* in a stock-shift fashion, as soon as the velocity shock exceeds the critical value v^*. The size of the change in (log) reserves in expression (7) is given by the horizontal distance between the pre-attack free-float locus FF corresponding to the critical level of reserves R^* and the post-attack line $F'F'$.

[**Figure 1 here**]

3 Reserve uncertainty

Suppose now that with a strictly positive probability δ the central bank can resort to a secondary reserve. The market finds out about the availability of this additional reserve only after the primary one has been exhausted. For simplicity, we assume that the size of the secondary reserve exceeds the critical amount R^*, so that the band is still fully credible after a first unsuccessful attack.

If no secondary reserve is available after the first one has been depleted, the post-attack exchange rate is given, as in section 2, by equation (5) corresponding to the $F'F'$ curve in figure 1. If, on the other hand, the monetary authority can still defend the band after exhausting its primary reserve, the post-attack exchange rate follows equation (3) with $m = \ln D$. This corresponds to curve ZZ in figure 1.

Since a rational attack can only take place at a level of the velocity shock v consistent with no *ex ante* expected jump in the exchange rate, it will be carried out at a level of

4

fundamentals $v^{**} > v^*$ in figure 1. If speculators launch an attack at v^{**}, with probability δ they make an *ex post* loss equal to the vertical distance QU. The exchange rate appreciates as a consequence of the attack since the demand for real money balances increases. With the complementary probability $(1 - \delta)$ the attack results in an *ex post* gain equal to the vertical distance PU. If $\delta = 1/2$, as we assume in figure 1, then v^{**} is determined by the requirement that $PU = QU$. More generally, the expected gain/loss has to be zero.

So the shadow floating exchange rate is a weighted average of the post-attack exchange rates under the two scenarios and is given by

$$s = \ln D + v - \delta \frac{e^{\lambda[\ln D + v - (\overline{m+v})]}}{\lambda}. \tag{8}$$

v^{**}, the new critical level for the velocity shock, satisfies

$$v^{**} = \bar{s} - \ln D + \delta \frac{e^{\lambda[\ln D + v^{**} - (\overline{m+v})]}}{\lambda}, \tag{9}$$

as the monetary authority intervenes only when s is at its upper boundary. Confronting equations (6) and (9) it is apparent that $v^{**} > v^*$, as long as $\delta > 0$. Uncertainty about a secondary reserve delays the timing of an attack, because it implies a positive downward risk. Correspondingly, the size $\bar{R}$ of the speculative attack is lower in the presence of reserve uncertainty. By equating (9) to (4) and remembering that v is continuous one obtains

$$\ln\left(D + \bar{R}\right) - \ln D = \frac{1}{\lambda} - \delta \frac{e^{\lambda\left[\ln D + v^{**} - (\overline{m+v})\right]}}{\lambda}. \tag{10}$$

It is well-known that a more depreciated intervention level $\bar{s}$ extends the expected survival time of a target zone by raising the level of the fundamental v at which an attack takes place. There is little a priori reason, though, to expect that the *increase* in the expected survival time stemming from a positive probability of survival δ, or equivalently the difference $v^{**} - v^*$, should be in any particular relationship with $\bar{s}$. A higher intervention parity $\bar{s}$ increases both v^{**} and v^* with an ambiguous effect on their difference. Put differently, a larger $\bar{s}$ increases both the *ex post* expected depreciation PU (see figure 1) in case a secondary reserve turns out not to be available and the expected appreciation QU in the opposite case. Yet, it can be proved that the second effect prevails.

Proposition 1 *For a given degree of reserve uncertainty, measured by the probability δ that a secondary reserve is available, the difference $v^{**} - v^*$ is an increasing function of the intervention parity $\bar{s}$.*

Proof. See appendix. ∎

Proposition 1 states that the last term in equation (9) is an increasing function of $\bar{s}$. Since this term enters with a negative sign in equation (10), the proposition implies that, in the presence of reserve uncertainty, the critical amount of reserves $\bar{R}$ necessary to support a band is a strictly decreasing function of $\bar{s}$. For a given degree of uncertainty $\delta > 0$, the stock of reserves necessary to ensure a positive expected survival time if the band is started at $s = \bar{s}$ tends to zero for $\bar{s}$ large enough. This result is new and applies only in the case of uncertain reserves. It relies only on the assumption that the speculative attack goes in the "right" direction; i.e. that the expected rate of currency depreciation is higher if the band collapses than if it survives. It does not depend either on the assumption of perfect credibility nor on that of a unilateral band[1].

Acknowledgements

---

[1]In the case of a two-sided target zone, the critical stock of reserves is an increasing function of the distance between the central parity and the upper parity $\bar{s}$ if $\delta = 0$, but a decreasing one if $\delta > 0$. A proof for this case is available on request from the author.

References

Krugman, P., 1979, A model of balance of payments crises, Journal of Money, Credit and Banking, 11, 311–325.

Krugman, P., 1997, Currency crises, Paper presented at NBER conference.

Krugman, P. and J. Rotemberg, 1992, Speculative attacks on target zones, in: P. Krugman and M. Miller, eds., Exchange Rate Targets and Currency Bands, Cambridge University Press, Cambridge: UK.

Appendix: Proof of proposition 1

From equations (6) and (9) the difference between the level of fundamentals at which the band collapses with and without reserve uncertainty, respectively v^{**} and v^*, is $v^{**} - v^* = \delta y$, where

$$y = \frac{e^{\lambda[\ln D + v^{**} - (\overline{m+v})]}}{\lambda}. \tag{11}$$

We need to show that y is an increasing function of the band size $\bar{s}$. Differentiating (9) with respect to $\bar{s}$ results in

$$\frac{dy}{d\bar{s}} = \frac{e^{\lambda[\ln D + v^{**} - (\overline{m+v})]}}{\lambda} \left(\frac{dv^{**}}{d\bar{s}} - \frac{d(\overline{m+v})}{d\bar{s}} \right). \tag{12}$$

The two terms in brackets can be obtained by differentiation of equations (9) and (4) and are given by

$$\frac{dv^{**}}{d\bar{s}} = 1 + \delta \frac{dy}{d\bar{s}} \tag{13}$$

and

$$\frac{d(\overline{m+v})}{d\bar{s}} = 1. \tag{14}$$

Replacing in equation (12) and rearranging results in

$$\frac{dy}{d\bar{s}} = \frac{e^{\lambda[\ln D + v^{**} - (\overline{m+v})]}}{(1-\delta)\lambda} > 0. \tag{15}$$

Figure 1: Speculative attack with reserve uncertainty.